ARITHMETIC

AVERAGES

Simple Average = $\frac{Sum\ of\ elements}{Number\ of\ elements}$

ArithmeticMean=$(a_1+a_2+a_3....a_n)/n$

Geometric Mean= $\sqrt[n]{a_1 a_2 \ldots . a_n}$

Harmonic Mean = $\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}}$

Fortwonumbersaandb
$\Longrightarrow$ AM=(a+b)/2
$\Longrightarrow$ GM = $\sqrt{ab}$
$\Longrightarrow$ HM = $\frac{2ab}{a+b}$

Concept: AM GM HM is always true. They will be equal if all elements are equal to each other. If I have just two values then GM^2 = AM x HM

Concept: Thesumofdeviation(D)ofeachelementwith respect to the average is 0

Median of a finite list of numbers can be found by arranging all the observations from lowest value to highest value and picking the middle one.

Mode is the value that occurs most often.

INTEREST

Amount=Principal+Interest SimpleInterest=PNR/100
Compound Interest = $P(1+\frac{r}{100})^n - P$

Population formula P' = $P\left(1 \pm \frac{r}{100}\right)^n$

Concept: SI and CI are same for a certain sum of money (P) at a certain rate (r) per annum for the first year. The difference after a period of two years is given by.

$\Rightarrow \Delta = \frac{PR^2}{100^2}$

Growth and Growth Rates
Absolute Growth = Final Value – Initial Value

Growth rate for one year period = $\frac{Final\ value - Initial\ value}{Initial\ value} \times 100$

SAGR or AAGR = $\left(\frac{Finla\ value - Initial\ Value}{Initial\ Value}\right)^{\frac{1}{no.of\ years}}$

Concept: If the time period is more than a year, CAGR < AAGR. This can be used for approximating the value of CAGR instead of calculating it.

PROFIT AND LOSS

$$\%\text{profit/Loss} = \frac{Selling\ Price - Cost\ Price}{Initial\ Value} \times 100$$

$$\%\ \text{Profit} = \left(\frac{Claimed\ Weight - Actual\ Weight}{Actual\ Weight} - 1\right) \times 100$$

$$\text{Discount}\% = \left(\frac{Marked\ price - Selling\ Price}{Marked\ Price} \times 100\right)$$

Concept: Effective discount after successive discount of a% and b% is $(a + b - \frac{ab}{100})$. Effective discount when you buy x goods and get y goods free is $\frac{y}{x+y} \times 100$.

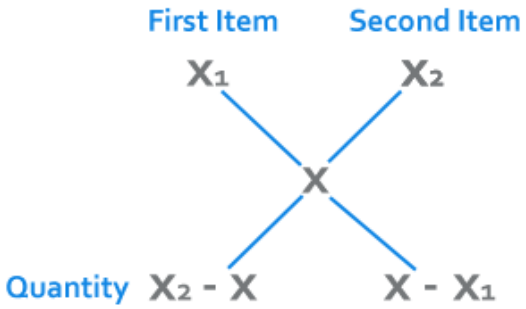
MIXTURES AND ALLIGATION

Successive Replacement – Where a is the original quantity, b is the quantity that is replaced and n is the number of times the replacement process is carried out, then

$$\frac{\text{Quantity of original entity after n operations}}{\text{Quantity of mixture}} = \left(\frac{a - b}{a}\right)^n$$

Alligation: The ratio of the weights of the two items mixed will be inversely proportional to the deviation of attributes of these two items from the average attribute of the resultant mixture

$$\Rightarrow \frac{\text{Quantity of first item}}{\text{Quantity of second item}} = \frac{x_2 - x}{x - x_1}$$



Ratio andProportion

CompoundedRatiooftworatiosa/bandc/disac/bd, **Duplicate ratio** of a: b is $a^2 : b^2$
Triplicate ratio of a : b is $a^3 : b^3$
Sub-duplicate ratio of a : b is $\sqrt{a} : \sqrt{b}$
Sub-triplicate ratio of a a : b is $\sqrt[3]{a} : \sqrt[3]{b}$
Reciprocal ratio of a : b is b : a.

Componendo and Dividendo

If $\frac{a}{b} = \frac{c}{d}$ & $a \neq b$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Four (non-zero) quantities of the same kind a, b, c, d are said to be in proportion if $\frac{a}{b} = \frac{c}{d}$.

Thenon-zeroquantitiesofthesamekinda,b,c,d..are saidtobeincontinued proportion if $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

Proportion

a,b,c,daresaidtobeinproportionif $\frac{a}{b}=\frac{c}{d}$

a,b,c,daresaidtobeincontinuedproportionif $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$

Concept: If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$ = k

- $\frac{a+c+e}{b+d+f}=k$
- $\frac{ap+qc+re}{pb+qd+rf}=k$
- $\frac{pa^n+qc^n+re^n}{pb^n+qd^n+rf^n}=k^n$

Given two variables x and y, y is **(directly) proportional** to x (x and y **vary directly**, or x and y are in **direct variation**) if there is a non-zero constant *k* such that y = kx. It is denoted by y α x

Two variables are **inversely proportional** (or **varying inversely**, or in **inverse variation**, or in **inverse proportion** or **reciprocal proportion**) if there exists a non-zero constant *k* such that y = k/x.

Time Speed and Distance

Speed = Distance / Time

1 kmph = 5/18 m/sec;
1 m/sec = 18/5 kmph

$\text{Speed}_{avg}=\frac{Total\ Distance\ Covered}{Total\ Time\ Taken}=\frac{d_1+d_2+d_3\ldots d_n}{t_1+t_2+t_3\ldots t_n}$

If the distance covered is constant then the average speed is Harmonic Mean of the values (s_1, s_2, s_3 …s_n)

⇒ $\text{Speed}_{avg}=\frac{n}{1/s_1+1/s_2+1/s_3\ldots 1/s_n}$

⇒ $\text{Speed}_{Avg}=\frac{(s_1+s_2)}{2}$ (for two speeds)

Concept: Given that the distance between two points is constant, then
- If the speeds are in *Arithmetic Progression*, then the times taken are in *Harmonic Progression*
- If the speeds are in *Harmonic Progression*, then the times taken are in *Arithmetic Progression*

For Trains, time taken = $\frac{\textbf{Total length to be covered}}{\textbf{Relative speed}}$

For Boats,
$\text{Speed}_{upstream}=\text{Speed}_{Boat}-\text{Speed}_{River}$
$\text{Speed}_{Downstream}=\text{Speed}_{Boat}+\text{Speed}_{River}$

$\text{Speed}_{Boat}=\left(\frac{Speed\ Downstream+Speed\ Upstream}{2}\right)$

$\text{Speed}_{River}=\left(\frac{Speed\ Downstream+Speed\ Upstream}{2}\right)$

For Escalators,
The difference between escalator problems and boat problems is that escalator can go either up or down.

Races & Clocks

Linear Races
Winner's distance = Length of race
Loser's distance = Winner's distance – (beat distance + start distance)

Winner's time = Loser's time – (beat time + start time)
Deadlock / dead heat occurs when beat time = 0 or beat distance = 0

Circular Races
Two people are running on a circular track of length L with speeds a and b in the same direction

- Time for 1st meeting = $\frac{L}{a-b}$
- They meet at a – b distinct points (reduced ratio)
- Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a}, \frac{L}{b}\right)$

Two people are running on a circular track of length L with speeds a and b in the opposite direction

- Time for 1st meeting = $\frac{L}{a+b}$
- They meet at a + b distinct points (reduced ratio)
- Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a}, \frac{L}{b}\right)$

Three people are running on a circular track of length L with speeds a, b and c in the same direction

- Time for 1st meeting = LCM $\left(\frac{L}{a-b}, \frac{L}{a-c}\right)$
- Time for 1st meeting at the starting point = LCM $\left(\frac{L}{a}, \frac{L}{b}, \frac{L}{c}\right)$

Clocks To solve questions on clocks, consider a circular track of length 360°. The minute hand moves at a speed of 6° per min and the hour hand moves at a speed of ½° per minute. The angle at H : M is given by |30H – 5.5M|

Concept: Hands of a clock coincide (or make 180) 11 times in every 12 hours. Any other angle is made 22 times in every 12 hours.

Time and Work

If a person can do a certain task in t hours, then in 1 hour he would do 1/t portion of the task.

A does a particular job in 'a' hours and B does the same job in 'b' hours, together they will take $= \frac{ab}{a+b}$
A does a particular job in 'a' hours more than A and B combined whereas B does the same job in 'b' hours more than A and B combined, then together they will take $\sqrt{ab}$ hours to finish the job.

Concept:If A does a particular job in 'a' hours and A&B together do the job in 't' hours, the B alone will take $\frac{at}{a-t}$ hours.
Concept:If A does a particular job in 'a' hours, B does the same job in 'b' hours and ABC together do the job in 't' hours, then

C alone can do it in $\frac{abt}{ab-at-bt}$ $hours$

Concept: If the objective is to fill the tank, then the*Inlet pipes*do**positivework**whereas the*Outletpipes*do **negativework**.If the objective is to empty the tank, then the*Outletpipes*do**positive work** whereas the pipes do negative work.

Quadratic and Other Equations

For a quadratic equation, $ax^2 + bx + c = 0$, its roots

- α or β= $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- Sum of roots= α+β=-$\frac{b}{a}$
- Product of roots = αβ=$\frac{c}{a}$

Discriminant

Condition	NatureofRoots
$\Delta < 0$	Complex Conjugate
$\Delta = 0$	Real and equal
Δ > 0andaperfectsquare	Rational and unequal
Δ > 0and not aperfectsquare	Irrationalandunequal

Concept:If c = a, then roots are reciprocal of each other
Concept: If b=0,then roots are equal in magnitude but opposite in sign.
Concept: Provided a, b and c are rational
⇒If one root is p + iq, other root will be p – iq
⇒If one root is p +$\sqrt{q}$, other root will be p – $\sqrt{q}$

Cubic equation $ax^3+bx^2+cx+d = 0$
⇒Sum of the roots = - b/a
⇒Sum of the product of the roots taken two a time = c/a
⇒Product of the roots = -d/a

Biquadratic equation $ax^4 + bx^3 + cx^2 + dx + e = 0$
⇒ Sum of the roots = - b/a
⇒ Sum of the product of the roots taken two a time = c/a
⇒ Sum of the product of the roots taken three a time = -d/a
⇒ Product of the roots = e/a

Inequalities

If a > b and c > 0,
⇒ a + c > b + c
⇒ a - c > b - c
⇒ ac > bc
⇒ a/c > b/c

If a, b ≥ 0, then $a^n > b^n$ and $1/a^n < 1/b^n$, where n is positive.

a < b and x > 0, then $\frac{a+x}{b+x} > \frac{a}{b}$
a>b and x>0, then

Modular Inequalities

|x- y| = |y - x|
|x. y| = |x| . |y|
|x+ y| < |x| + |y|
|x+ y| > |x| - |y|

Quadratic Inequalities
(x–a)(x–b)>0 {a <b}
⇒(x < a) U (x > b)

$(x-a)(x-b)<0$ $\{a>b\}$

$\Rightarrow a < x < b$

For any set of positive numbers: AM≥GM≥HM

$\Rightarrow \frac{a_1+a_2+ \ldots.+a_3}{n} \geq (a_1.a_2.\ldots\ldots a_n)^{1/n}$

If a and b are positive quantities, then $\frac{a+b}{2} \geq \sqrt{ab}$