

Co-ordinate Geometry

Distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

If a point $R(x, y)$ divides $P(x_1, y_1)$ and $Q(x_2, y_2)$ internally in the ratio of $m:n$, the coordinates of R i.e. (x, y) are given by

$$= \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}$$

If a point $R(x, y)$ divides $P(x_1, y_1)$ and $Q(x_2, y_2)$ externally in the ratio of $m:n$, the coordinates of R i.e. (x, y) are given by

$$= \frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}$$

Concept: The X axis divides the line joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio of $y_1 : y_2$

Concept: The Y axis divides the line joining $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the ratio of $x_1 : x_2$

Slope(m) of a line is the tangent of the angle made by the line with the positive direction of the X-Axis.

For a general equation $ax + by + c = 0$; slope (m) = $-a/b$. For a line joining two points, $P(x_1, y_1)$ and $Q(x_2, y_2)$, the slope(m) is $= \frac{y_2 - y_1}{x_2 - x_1}$

Slope(m)	Type of line	Angle with X- Axis
> 0 (+ive)	Rising	Acute
0	Parallel to X-Axis	0°
< 0 (-ive)	Falling	Obtuse
infinite	Parallel to Y-Axis	90°

Equation of a line parallel to X-axis is $y = a$ {of X-Axis is $y = 0$ }

Equation of a line parallel to Y-Axis is $x = a$ {of Y-Axis is $x = 0$ }

The intercept of a line is the distance between the point where it cuts the X-Axis or Y-Axis and the origin. Y- Intercept is often denoted with the letter 'c'.

Equation of a line

General form: $ax + by + c = 0$

Slope Intercept Form: Slope is m , y-intercept is c

$$\Rightarrow y = mx + c$$

Slope Point Form: Slope is m , point is (x_1, y_1)

$$\Rightarrow y - y_1 = m(x - x_1)$$

Two Point Form: Two points are (x_1, y_1) and (x_2, y_2)

$$\Rightarrow (y - y_1) = \left[\frac{y_2 - y_1}{x_2 - x_1} \right] (x - x_1)$$

Two Intercept Form: X-intercept is a , Y-intercept is b .

$$\Rightarrow \frac{x}{a} + \frac{y}{b} = \pm 1 \text{ OR } bx + ay = ab$$

Acute angle between two lines with slope m_1 and m_2 is given by

$$\Rightarrow \tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

$$\Rightarrow \text{For parallel lines, } \theta = 0^\circ; m_1 m_2 = 1$$

$$\Rightarrow \text{For perpendicular lines, } \theta = 90^\circ; m_1 m_2 = -1$$

Distance of a point $P(x_1, y_1)$ from a line $ax + by + c = 0$

$$\Rightarrow d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

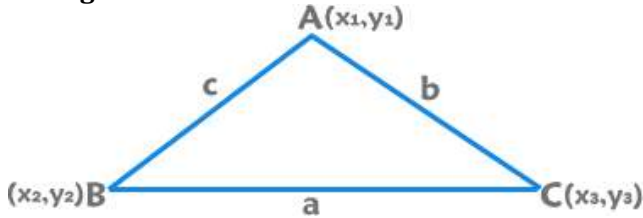
$$\Rightarrow \text{From origin, } d = \left| \frac{c}{\sqrt{a^2 + b^2}} \right|$$

Distance between two parallel lines, $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$

$$\Rightarrow d = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

Concept: If we know three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ of a parallelogram, the fourth point is given by $\Rightarrow (x_1 + x_3 - x_2, y_1 + y_3 - y_2)$

Triangle



The vertices are $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$

$$\text{Incenter} = \left\{ \frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right\}$$

$$\text{Centroid} = \left\{ \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right\}$$

$$\text{Area} = \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Circle

General Equation: $x^2 + y^2 + 2gx + 2fy + c = 0$

\Rightarrow Centre is $(-g, -f)$ and radius $= \sqrt{g^2 + f^2 - c}$

\Rightarrow Centre is (h, k) and radius is r

$$\Rightarrow \sqrt{(x - h)^2 + (y - k)^2} = r$$

Centre is origin and radius is r

$$\Rightarrow x^2 + y^2 = r^2$$