SEQUENCE & SERIES

(Ref: FM-QAH2022020)

A) Arithmetic Progression (A.P.)

TYPE I

1. In an AP the first term is 15 and the common difference is 4. Find the fifteenth term.

2. The third term of an AP is 25 and the 23rd term is 65. Find the 63rd term.

3. Which term of the arithmetic progression 2, 6, 10,... is 118?

4. What is the 14th term of an arithmetic progression whose first term is equal to its common difference and whose 3rd term is 12?
   a) 36 b) 40 c) 56 d) 60

5. Thirteen times the thirteenth term of an arithmetic progression is equal to seven times the seventh term of the arithmetic progression. What is the twentieth term?

6. The first term of an arithmetic progression is 5 and the common difference is 6. The nth term is 245. Find the value of n.

7. Consider the two sequences
   3, 6, 9, 12, 15, …108 and 4, 8, 12, 16, … 136.
   Find the number of terms common to both the sequences.

8. If Pth term of an A.P. in Q and Qth term of same A.P. is P. Find (P+Q) th term.

TYPE II

9. Find the sum up to 50 terms of an AP whose first term is 50 and common difference is -1.

10. Find the sum up to 25 terms of an AP whose third term is 2/5 and 23rd term is 88/5.

11. If the 7th term of an AP is 90, what is the sum of the first 13 terms?

12. Find the number of terms and the sum of the terms respectively of the arithmetic progression 50, 47, 44, …. 5.
    a) 15, 440 b) 16, 420 c) 16, 440 d) 15, 420

13. The sum to n terms of an arithmetic progression is $2n^2+n$. Find the n' term.
    a) 4n - 3 b) 4n – 2 c) 4n + 2 d) 4n + 3

14. Find the sum of all the two-digit numbers which leave a remainder of 1 when divided by 5.
    a) 936 b) 963 c) 954 d) 945

15. (a) The sum of the first 29 terms of an arithmetic progression is 0. Which of the following terms must be 0?
    a) 18th b) 19th c) 15th d) 16th

16. Find the least value of the number of terms of the series 24, 20, 16, 12,… for which the series has the maximum sum.
    a) 5 b) 6 c) 7 d) 8

17. How many terms of the A.P. 2, 4, 6, . . . must be taken so that the sum is 132?
    a) 10 b) 11 c) 21 d) 12

18. Find the sum of all the integers from 1 to 300 that are divisible either by 3 or 5.

19. In an AP
    a) 15th term is 25 and 23rd term is 35. Find sum upto 61 terms.
    b) 5th term is 75 and 10th term is 120. Find sum upto 15 terms and 25 terms.
    c) If sum upto 20 & 30 terms is 610 and 1365. Find sum upto 50 terms.
    d) Sum upto 20 terms is 440 and sum of next 20 terms is 1240. Find sum upto 50 terms.

20. S = {1, 2, 3, 4, 5, ……, 1000}
    How many A.P's can be formed from the elements of 's' that start with 1 and end with 1000 and have atleast three elements?

B) Geometric Progressions (G.P.)

TYPE I

21. In a GP the first term is 5 and the common ratio is 3. What is the 6th term?

22. In a GP the first term is 3 and the common ratio is 2. What is the sum of the first 6 terms?

23. In a geometric progression, the 3rd term is 9 and the 7th term is 729. What is the 4th term?
    a) 81 b) -81 c) 27 d) -27

24. Find the common ratio of G.P. If the 3rd term is 4.5 and 6th term is 243/16. Also find 5th term.

TYPE II

25. (a) Find the sum of the first 4 terms of a geometric progression whose first term is 6 and whose common ratio is 3.
    a) 120 b) 180 c) 210 d) 240

    (b) What is the sum of the first 7 terms of a geometric progression whose first term is 1 and 4th term is 8?
    a) 129 b) 128
    c) 127 d) None of these

26. In a GP the first term is 11 and the last term is 2816. The sum of all the terms is 5621. Find the common ratio.

27. In an infinite GP, every term is 7 times the sum of all the terms that follow. If the sum of the first and the second terms is 18, find the terms of the GP.

28. Find the common ratio of a geometric progression. whose first term is 3, last term is 2187 and the sum of the terms is 3279.

29. The product of three terms in geometric progression is 1728 and the sum of the products of two of them taken at a time is 1032. Find the smallest of the three terms.

30. The sum of the first eight terms of a geometric progression is 510 and the sum of the first four terms of the geometric progression is 30. Find the first term of the geometric progression, given that it is positive.
a) 2 b) 4 c) 6 d) 8

31. The number of bacteria in a colony doubles every minute. If there are 1024 bacteria after 5 minutes, find the number of bacteria present initially.

32. The sum of the first 2012 terms of a geometric progression is 200. The sum of the first 2024 terms of the progression is 380. The sum of the first 6036 terms of the progression is

33. Sum of an infinite G.P. is 3/2. Also the sum of 1st, 3rd, 5th, 7th term of same infinite G.P. is 9/8. Find r.

34. In a GP, $T_5 = 3$. What is the product of the first nine terms?

35. In a GP with even number of terms, the ratio of the sum of the odd-numbered terms to the sum of the even-numbered terms is 1 : 5. Find r.

36. Sum of three numbers in GP is 13 and their product is 27. Find the numbers.

37. In a G.P. of a +ve natural numbers, any term is one move than the sum of all the terms before it. How such series are possible?

C) Harmonic progression

38. (a) If the second and the twelfth terms of a Harmonic Progression are 1/5 and 1/35 respectively, find the 27th term of the series.
(b) Find the harmonic mean of the terms 4/3, 4/7, 4/11, 4/15.

D) Arithmetic-Geometric Progression

39. (a) Find the value of $1 + 2x + 3x^2 + 4x^3 + \ldots \infty$, where x = 1/3.
(b) Find the value of $1 + 3x + 6x^2 + 10x^3 + 15x^4 +$ where x = 3/4

40. Find the value of 8 + 88 + 888 + ...n terms.

41. Find the value of $1 + \frac{4}{6} + \frac{9}{36} + \frac{16}{216} + \frac{25}{1296} \ldots . \infty$.

E) Some Special Series

42. A) $\frac{1}{5\times7} + \frac{1}{7\times9} + \frac{1}{9\times11} + \cdots . \infty$
B) $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{9}\right)\left(1+\frac{1}{81}\right)\left(1+\frac{1}{6561}\right)$

43. A) If $t_n = (5^n)^{1/2^n}$, find the product of infinite terms of the series.
B) 1+(1+2)+(1+2+3) + …….. + (1+2+3+ ….. +n)
C) Find the sum of series upto n terms
3, 7, 13, 21, 31, …….

44. Find the sum upto 1st terms of the series
a) (1 × 2) + (2 × 3) + (3 × 4) + (4 × 5) + ……
b) (1×3) + (2 × 4) + (3 × 5) + (4 × 6) + ……
c) (1^2 × 2) + (2^2 × 3) + (3^2 × 4) + ……

45. Find the value of (50×1)+(49×2)+(48×3)+…. (1×50).

46. Find the sum of the first 10 terms of the series $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + \sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}} + \sqrt{1+\frac{1}{4^2}+\frac{1}{5^2}} + \cdots + \sqrt{1+\frac{1}{10^2}+\frac{1}{11^2}}$
a) $7\frac{7}{11}$ b) $8\frac{8}{11}$ c) $9\frac{9}{11}$ d) $10\frac{10}{11}$

47. Find the value of $-1^2 + 2^2 - 3^2 + 4^2 - 5^2 + 6^2 + \ldots -19^2 + 20^2$

48. A ball is dropped from a height of 1250 m. It rebounds to four-fifths of the height from which it falls. If it continues to fall and rebound this way, how much distance does the ball cover totally before coming to rest? (in m)

49. Find the 280th term of the series a, b, b, c, c, c, d, d, d, d, ……

50. 1,2,2,4,4,4,4,8,8,8,8,8,8,8,8, …… find the 1030th term.

51. $t_{n+1} = t_n - t_{n-1}$, n > 1 and $t_1 = 10$, $t_2 = 7$
i) Find t_{100}
ii) Find $t_1 + t_2 + t_3 + \ldots. + t_{50}$

52. $t_n = \frac{t_{n-1}+t_{n+1}}{2}, n > 1$ and $t_1 = 1, t_2 = 3$
i) Find t_{100}
ii) Find $t_1 + t_2 + t_3 + \ldots. + t_{100}$