

INEQUALITIES AND MODULUS

(Ref: FM-QAH2022017)

I. Basics of Inequalities

- (a) The set of all real numbers lying between 6 and 9 is represented as
a) [6, 9] b) (6, 9] c) (6, 9) d) [6, 9]
- (b) The set of all real numbers lying between 2 and 7 including 5 is represented as
a) (2, 7) b) [2, 7) c) [2, 7] d) (2, 7]
- (c) The set of all real numbers from 5 to 8 is represented as
a) [5, 8] b) (5, 8) c) [5, 8) d) (5, 8]
- If $p > q$ and $r > 0$, then which of the following statements is/are true?
a) $p + r > q + r$ b) $p - r > q - r$
c) $pr > qr$ d) All the above
- If $p < q$ and $r < 0$ then which of the following is true?
a) $pr < qr$ b) $p/r < q/r$
c) $pr > qr$ d) None of these
- All real numbers less than or equal to 4 are included in
a) $(-\infty, 4]$ b) $(-\infty, 4)$
c) $(-\infty, -2) \cup (4, \infty)$ d) $(4, \infty)$

II. Solving Inequalities

- Solve the inequations:
(i) $10x + 20 < 8x + 16$
 $12x + 10 < 15x + 25$
(ii) $8x + 15 > 7x + 10$
 $6x + 9 < 2x + 21$
- Solve the inequations:
(i) $x^2 - 2x - 15 \leq 0$
(ii) $x^2 - 5x + 6 > 0$
- Solve the inequations:
(i) $x^2 + 12x + 37 > 0$
(ii) $x^2 - 11x + 31 < 0$
- Solve:
(i) $\frac{(x+5)(x+3)}{x+4} \geq 0$
(ii) $\frac{5x+11}{x-2} \geq 3$
- Solve the inequations:
(i) $(x+2)(x-3)(x+4)(x+5) \leq 0$
(ii) $(2-x)^5(x+5)^7(x+10)^4 > 0$
- Solve for x: $\frac{x^2-8x+12}{2x^2+7x-4} < 0$
- a) $(x-5)^2(x+2)x \leq 0$
b) $(x+1)^2(x+2)x < 0$

- $\frac{(x+3)(x^2-4)}{(x-1)^2} \leq 0$
- $\frac{(x-5)^2(x+8)}{x-7} \geq 0$
- $\frac{x+8}{(x-7)(x+10)^2} > 0$
- $\frac{(x-5)^2x^3}{(x^2-4)(x+4)} \leq 0$
- $(x+2)^3(x-5) \leq 0$

- a) Solve $\frac{1}{x} > \frac{1}{2}$
b) Solve $\frac{1}{x-2} > \frac{1}{5}$
- How many integer value of x will not satisfy the inequality $-\frac{1}{3} \leq \frac{1}{x} \leq \frac{1}{4}$?
- The number of integral values of x, for which $\frac{x-5}{x+1} < 0$ is
- The number of integral values of x, which the inequation $\frac{x-2}{x+3} > 2$ is satisfied, is

Modulus

I) Basics of Modulus

- When $x < 0$, then $|x| =$
a) 0 b) x c) $\pm x$ d) $-x$
- Solve Modulus
(A) $|x| = 5$
(B) $|x - 2| = 5$
(C) $|2x + 4| = 10$
(D) $|3x - 2| = 10$

II) Modulus and Inequality Mix

- Solve followings
(A) $|x| < 5$
(B) $|x| > 5$
(C) $|x - 3| < 10$
(D) $|x + 5| \leq 2$
- Solve
(i) $|9x + 7| \leq 25$
(ii) $|13x + 29| \geq 68$
- Find the range of all values of x if $|2x + 3| \geq 11$.
a) (-7, 4) b) [-7, 4]
c) (-7, -4) d) $(-\infty, -7] \cup [4, \infty)$
- If $|5x + 3| > 18$ and $x > 0$, then $x >$
- Solve $|x^2 - 5x| < 6$

23. How many integral values of x will not satisfy $x^2 - 2|x| - 8 \geq 0$?
24. Find no. of solutions for
 i) $x^2 - 5|x| + 6 = 0$
 ii) $x^2 - 3|x| - 4 = 0$
 iii) $x^2 + 5|x| + 6 = 0$
25. For how many integral values of x , is the expression $|x^2 - 9x + 18| > x^2 - 9x + 18$ valid?
26. Find possible values of x in,
 i) $||x - 3| - 2| + 1| = 4$
 ii) $||x - 2| - 1| - 3| = 1$
27. The number of distinct solutions of the equation $|2x - 3x - 3| = 15$ is
28. The number of integral solution of the inequality $|x - 4| + |x - 7| < 2$ is
29. The number of integral values of x that satisfy the inequation $|x - 2| + |x - 3| \leq 5$ is
30. Solve for x :
 i) $|x - 3| + |x + 5| = 7$
 ii) $|x - 3| + |x + 5| = 8$
 iii) $|x - 2| + |x + 4| = 9$
31. How many integral values of x satisfy the equation $|x-6|+|x-3|+|x+3|+|x+6|=18$?

III) Maxima/Minima

32. i) $f(x) = 25 + |5x + 4|$. Find the minimum value of $f(x)$ and the value of x for which it is minimum.
 ii) $f(x) = 20 - |3x + 9|$. Find the maximum value of $f(x)$.
33. i) $f(x) = |x - 2| + |x - 3| + |x - 5|$. Find the minimum value of $f(x)$ and also find the value of x at which $f(x)$ attains its minimum value.
 ii) $f(x) = |x - 3| + |x - 5|$. Find the minimum value of $f(x)$ and also find the value of x at which $f(x)$ attains its minimum value.
34. Find the minimum and maximum value of $\frac{x^2 - x + 1}{x^2 + x + 1}$, x being real number.
35. $|2x - 5| \leq 7$ and $|13y - 6| \leq 18$. Find the maximum value of $|x| - |y|$.
36. If $-5 < x < 7$ and $-8 < y < 3$, then find the range of xy .
37. If a, b, c are positive real numbers and $a+b+c = 12$. Find maximum value of $(a+1)(b+2)c$.

38. If x, y are positive real numbers and $2x + 3y = 15$, find the maximum value of x^2y^3 .
39. If a, b are positive real number number $ab = 6$. Find minimum sum of $2a + 3b$.
40. If x, y are positive real numbers and $xy^2 = 27$, find the minimum value of $32x + y$.
41. If $ac = bd = 2$, then the minimum value of $a^2 + b^2 + c^2 + d^2$ is
42. If x, y, z are distinct positive real numbers then find the range of $\frac{xy(x+y)+yz(y+z)+zx(z+x)}{xyz}$.
43. If x, y , and z are non-zero real numbers, find the minimum value of $\frac{9(x^2+y^2)}{4z^2} + \frac{9(y^2+z^2)}{4x^2} + \frac{9(z^2+x^2)}{4y^2}$.
44. If a, b and c are three positive real numbers then find the minimum value of $\frac{(5a^2+a+5)(7b^2+b+7)(9c^2+c+9)}{57abc}$.
45. If a, b and c are the sides of the triangle, what is the range of $\frac{a^2+b^2+c^2}{ab+bc+ca}$?
46. If x, y and z are non-zero real numbers, find the minimum value of $\frac{(x^2+y^2)(y^2+z^2)(z^2+x^2)}{4x^2y^2z^2}$.
47. If $x^2 + y^2 + z^2 = 24$ then what is the minimum value of $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2}$?
48. Let x, y, z be three positive real numbers such that $x - y = z - x$ and $xyz = 8$. Find the minimum value.
49. If x, y, z are positive numbers and $xyz = 512$. Which of the following is not a possible value of $xy+yz+zx$?
 a) 326 b) 433 c) 291 d) 96
50. If a, b and c are positive numbers and $a^3 + b^3 + c^3 = 81$, then the maximum value of abc is
51. If a, b, c, d and e are positive numbers such that $abcde = 81$, then the minimum value of $(1+a)(1+b)(1+c)(1+d)(1+e)$ is
52. The range of values of a for which $-x^2 + 3ax + 5a + 1 < 0$ is
 a) $[-9/2, -2]$ b) $(\infty, -2) \cup (-2/9, \infty)$
 c) $(-2, -2/9)$ d) $(-\infty, \infty)$
53. Which is greater: 50^{51} or 51^{50} ?

Answer Key

1. A) c
B) b
C) a
2. D
3. C
4. A
5. i) $-5 < x < -2$
ii) $x < 5$
6. i) $[-3, 5]$
ii) $(-\infty, 2) \cup (3, \infty)$
7. i) $x \in \mathbb{R}$
ii) No Solution
8. i) $[-5, -4] \cup [-3, \infty]$
ii) $(-\infty, -17/2] \cup [2, \infty)$
9. i) $[-5, -4] \cup [2, 3]$
ii) $(-\infty, -10) \cup (-5, 2)$
10. $(-4, 1/2) \cup (2, 6)$
11. a) $[-2, 0] \cup \{5\}$
b) $(-2, 0) - \{-1\}$
c) $-2 \leq x \leq 2$ or $x \leq -3, x \neq 1$
d) $x > 7$ or $x \leq -8, x = 5$
e) $x > 7$ or $x < -8, x \neq -10$
f) $-4 < x < -2$ or $0 < x < 2, x = 5$
g) $-2 \leq x \leq 5$
12. a) $0 < x < 2$
b) $2 < x < 7$
13. 6
14. $(-1, 5)$

15. $(-8, -3)$
16. D
17. A) $x = \pm 5$
b) $x = -3$ or 7
c) $x = 3$ or -7
d) $x = 4$ or $-8/3$
18. A) $-5 < x < 5$
b) $x > 5$ or $x < -5$
c) $-7 < x < 13$
d) $-7 \leq x \leq -3$
19. i) $-32/9 \leq x \leq 18/9$
ii) $(-\infty, -97/3] \cup [3, \infty)$
20. D
21. 3
22. $(-1, 2) \cup (3, 6)$
23. 7
24. i) four
ii) two
iii) No solution
25. Two
26. i) $x = -2$ or 8
ii) $x = 7, -3, 5, -1$
27. Two
28. No solution
29. Six

30. i) No solution
ii) $-5 \leq x \leq 3$
iii) $x = 3.5$ or -5.5
31. 7
32. i) 25, $-4/5$
ii) 20
33. i) 3, 3
ii) 2, $[3, 5]$
34. $1/3, 3$
35. 6
36. $(-56, 40)$
37. 125
38. 243
39. 12
40. 18
41. 8
42. $[6, \infty)$
43. 13.5
44. 55
45. $[1, 2)$
46. 2
47. $3/8$
48. 2
49. D
50. 27
51. 1024
52. C
53. 50^{51}

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