Algebra Basics:

Variables : A variable is a letter used to stand for a number. The letters **e** and *i* have special values in algebra and are usually not used as variables

Algebraic expressions: Variables are used to change verbal expressions into algebraic expressions, that is, expressions that are composed of letters that stand for numbers. Eg: xyz, X +1, 2(x-1) +3x

Algebraic equations: If a Algebraic expression is assigned a value(either constant or expression), it forms a value.

Important Key Words

Add

Addition: The team needed the addition of three new players ... Sum: The sum of 5, 6, and 8 ...

Total: The total of the last two games ... Plus: Three chairs plus five chairs ... Increase: Her pay was increased by 30 rupees ... Note:

For Age/Time/Clock Problem

: **<Time Period> after/from now/hence** Important Key Words

Substract:

Substracted: Four is substracted from a number difference: The difference of 5, and a number ... Less than: Three less than a number ... Decreased by: A number is decreased by Taken away: Five apples are taken away from a pack. For Age/Time/Clock Problem : <Time Period> before/ago

Multiply

Product: The product of 3 and 6 is ...
Of: One-half of the people in the room ...
Times: Six times as many men as women ...
At: The cost of five yards of material at \$9 a yard is ...
Total: If you spend \$20 per week on gas, what is the total for a two-week period ...

Twice: *Twice the value of some number* ... (multiplying by 2) **Thrice** *:Thrice the value of some number* ... (multiplying by 3)

Divide

Quotient: The final quotient is ...
Divided by: Some number divided by 5 is ...
Divided into: The coins were divided into groups of ...
Ratio: What is the ratio of ...
Half: Half of the cards were ... (dividing by 2)
One-third: one-third of a number...(divided by 3)

Give the algebraic expression for each of the following.

- 1. the sum of a number and 5
- 2. the number minus 4
- 3. six times a number
- 4. x divided by 7

- 5. three more than the product of 2 and x
- 6. Twice a number decreased by four
- 7. Thrice a number reduced by one-third of another number
- 8. A number increased by 10%
- 9. Three years ago the age of ramesh was
- 10. At what time the train will reach the station.

1. Convert the following statements into equations.

(a) 5 added to a number is 9.

(b) 3 subtracted from a number is equal to 12.

(c) 5 times a number decreased by 2 is 4.

(d) 2 times the sum of the number x and 7 is 13.

2. A number is 12 more than the other. Find the numbers if their sum is 48.

3. Twice the number decreased by 22 is 48. Find the number.

4. Seven times the number is 36 less than 10 times the number. Find the number.

5. 4/5 of a number is more than 3/4 of the number by 5 . Find the number.

6. The sum of two consecutive even numbers is 38. Find the numbers.

7. The sum of three consecutive odd numbers is 51. Find the numbers.

8. Rene is 6 years older than her younger sister. After 10 years, the sum of their ages will be 50 years. Find their present ages.
9. The length of a rectangle is 10 m more than its breadth. If the perimeter of rectangle is 80 m, find the dimensions of the rectangle.

10. In a class of 42 students, the number of boys is 2/5 of the girls. Find the number of boys and girls in the class.

11. Among the two supplementary angles, the measure of the larger angle is 36° more than the measure of smaller. Find their measures.

12. My mother is 12 years more than twice my age. After 8 years, my mother's age will be 20 years less than three times my age. Find my age and my mother's age.

13. In an isosceles triangle, the base angles are equal and the vertex angle is 80°. Find the measure of the base angles.

14. Adman's father is 49 years old. He is 5 years older than four times Adman's age. What is Adman's age?

15. Divide 36 into two parts in such a way that 1/5 of one part is equal to 1/7 of the other.

16. The Sum of a two digit number and the digit formed by reversing the digits at unit place is 33. Find the numbers.

Polynomial: Polynomial comes from *poly-* (meaning "many") and *-nomial* (in this case meaning "term") ... so it says "many terms". A polynomial can have:

constants (like 3, -20, or 1/2)

variables (like x and y)

 $\underline{\text{exponents}}$ (like the 2 in y²), but only 0, 1, 2, 3, ... etc are allowed

Term: The expression which only have multiplication and division operand.

Like Terms: Terms having same degree in order of variables are called like terms.

Unlike Terms: Terms having different degree of variables.

A monomial is a polynomial that consists of exactly one term Ex. $56x^{23}$

A **binomial** is a polynomial that consists of exactly two terms. Ex. 5x-7

A **trinomial** is a polynomial that consists of exactly three terms Ex. $4x^{6}+15(x-8)+1$.

Polynomials in **one variable** are algebraic expressions that consist of terms in the form ax^n , where n is a non-negative (*i.e.* positive or zero) integer and "a" is a real number and is called the **coefficient** of the term.

The **degree** of a polynomial in one variable is the largest exponent in the polynomial.

Polynomials in two variables are algebraic expressions consisting of terms in the form ax^ny^m . The degree of each term in a polynomial in two variables is the sum of the exponents in each term and the **degree (m+n)** of the polynomial is the largest such sum.

0-degree polynomials are called **constants**. The values of constants don't change, so they're used to describe quantities that don't change.

1st-degree polynomials are called **linear** polynomials. They are used to describe quantities that change at a steady rate. They are also used in many one-dimensional geometry problems involving length.

2nd-degree polynomials are called **quadratic** polynomials. They are used to describe quantities that change with some amount of acceleration or deceleration. They are also used in many two-dimensional geometry problems involving area.

3rd-degree polynomials are called **cubic** polynomials. They are used in many three-dimensional geometry problems involving volume.

Perform the indicated operation for each of the following.

- 1. Add $6x^5-10x^2+x-45$ to $13x^2-9x+4$.
- 2. Subtract 5x³-9x²+x-35from x²+x+1
- 3. Multiply $4x^2(x^2-6x+2)$
- 4. Multiply (3x+5)(x-10)

Factoring Polynomials

 The first method for factoring polynomials will be factoring out the **G.C.F**/H.C.F and using reverse Distributive Law.
 Factoring by Grouping the Like term and then using distributive law.

Factoring Quadratic Polynomials by splitting the middle term.
 Using Identities:

 $a^{2}+2ab+b^{2}=(a+b)^{2}$ $a^{2}-2ab+b^{2}=(a-b)^{2}$ $a^{2}-b^{2}=(a+b)(a-b)$ $a^{3}+b^{3}=(a+b)(a^{2}-ab+b^{2})$ $a^{3}-b^{3}=(a-b)(a^{2}+ab+b^{2})$

Factorize:

- 1. $x^{3}y^{2}+3x^{4}y+5x^{5}y^{3}$
- 2. $9x^{2}(2x+7)-12x(2x+7)$
- 3. $x^{5}-3x^{3}-2x^{2}+6$
- 4. $x^{2}+2x-15$
- 5. x²-10x+24
- 6. x²+6x+9
- 7. x²+5x+1
- 8. $3x^2+2x-83$
- 9. 5x²-17x+65
- 10. 4x²+10x-6
- 11. x²-20x+100
- 12. 25x²-9
- 13. 8x³+1
- 14. 3x⁴-3x³-36x²
- 15. x⁴-25
- 16. x⁴+x²-20

Reduce the following rational expression to lowest terms.

(1.	$\frac{x2-2x-8}{x2-9x+20}$
2.	x2-25 5x-x2
3.	$\frac{x7+2x6+x5}{x3(x+1)8}$
4.	$\frac{x2-5x-14}{x2-3x+2} \times \frac{x2-4}{x2-14x+49}$
5.	$\frac{m^2-9}{m^2+5m+6} \div \frac{3-m}{m+2}$
6.	$\frac{y2+5y+4}{5y2+4}$

Zeroes of Polynomial:

- 1. The value/values of the variable for which the expression has a value zero in case of equation.
- 2. The set of values for which the expression holds the given condition/inequality.

Linear equation:

Polynomial of degree one written in the form:

ax+b=f(y) where a and b are real numbers.

and x is a independent variable and y is dependent variables.

Note: It can also be seen as the set of co-ordinates(x,y) satisfying a line in Cartesian plane.

Solve:

- 1. 3(x+5)=2(-6-x)-2x3
- 2. m-23+1=2m
- 3. 52y-6=10-y
- 4. 2z+3=3z-10+2
- 5. The taxi driver charges Rs. 30 for the first Km and Rs. 15 per km thereafter. If he has a passenger for 40 km, how much should he charge?
- The area of a rectangle is directly proportional to it's length. What will be the change in area if it's length increases by 66.66%.
- 7. The length of a rectangle is twice its breadth. If the perime is 72 metre, find the length and breadth of the rectangle.
- 8. The Saving of Ramesh is 40% of it's expenditure. If ne expends Rs 400, which is one third of his earning, Find his earning?
- 9. Aaron is 5 years younger than Ron. Four years later, Ron will be twice as old as Aaron. Find their present ages.
- 10. Robert's father is 4 times as old as Robert. After 5 years, father will be three times as old as Robert. Find their present ages.

Pair of Linear Equations:

If two or more linear equations have the two same variables, they are called a pair of linear equations in two variables.

- Ex.
 - I. $a_1x + b_1y + c_1 = 0$
 - II. $a_2x + b_2y + c_2 = 0$

Note:

- a1/a2≠b1/b2, the pair of linear equations is consistent. This means there is unique solution for the given pair of linear equations. The graph of the linear equations would be two intersecting lines.
- a1/a2=b1/b2≠c1/c2, the pair of linear equations is inconsistent. This means there is no solution for the given pair of linear equations. The graph of linear equations will be two parallel lines.
- a1/a2=b1/b2=c1/c2, the pair of linear equations is dependent and consistent. This means there are infinitely many solutions for the given pair of linear equations. The graph of linear equations will be coincident lines. There are following three methods under Algebraic method

to solve the above system.

(i) Substitution method

(a) Find the value of one variable, say y in terms of x or x in terms of y from one equation.

(b) Substitute this value in second equation to get equation in one variable and find solution.

(c) Now substitute the value/solution so obtained in step (b) in the equation got in step (a).

(ii) Elimination Method

- (a) If coefficient of any one variable are not same in both the equation multiply both the equation with suitable non-zero constants to make coefficient of any one variable numerically equal.
- (b) Add or subtract the equations so obtained to get equation in one variable and solve it.
- (c) Now substitute the value of the variable got in the above step in either of the original equation to get value of the other variable.
- (iii) Cross multiplication method

For the pair of Linear equations intwo variables: $a_1x + b_1y + C_1 = 0$

$$a_2x + b_2y + c_2 = 0$$





Solve it to get the solution, provided $a_1b_2 - a_2b_1 \neq 0$

Questions:

- 1. Solve:
 - a) 5x-4y+8=0 and 7x+6y-9=0
 - b) 3x-4y=5 and 4x-3y =5
 - c) x/2 + y/3 = 10 and x/4 + y/5 = 15
 - d) 0.25 x +.66y =7 and 0. 66x + 0.25y =7
- Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." Find their present ages.
- 3. The cost of 2 kg apples and 1 kg grapes was found to be Rs. 160. After a month, the cost of 4 kg apples and 2 kg grapes is Rs. 300.
- 4. 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.
- 5. The middle digit of a number between 100 and 1000 is zero and the sum of the other digit is 13. If the digits are reversed, the number so formed exceeds the original number by 495. Find the number.
- 6. Once a mule and a donkey were talking. The mule said, "I am carrying more sacks than you. In fact, if you give me one of your sacks, then I would have twice as many as you. If I give you a sack, our loads would be equal." How many sacks was each animal carrying?
- A man buys postage stamps of denominations 25 paise and 50 paise for ` 10. He buys 28 stamps in all. Find the number of 25 paise stamps bought by him.
 - 8. The sum of the digits of a two digit number is 5. The digit obtained by increasing the digit in ten's place by unity is one-eighth of the number. Find the number.
 - If the numerator and denominator of a fraction are increased by 2 and 1 respectively, it becomes 3/4. If the numerator and denominator are decreased by 2 and 1 respectively, it becomes 1/2. Find the fraction.
 - 10. Six years hence a man's age will be three times his son's age, and three years ago he was nine times as old as his son. Find their present ages.
 - 11. Vijay had some bananas, and he divided them into two lots A and B. He sold the first lot at the rate of `2 for 3 bananas and the second lot at the rate of `1 per banana, and got a total of `400. If he had sold the first lot at `1 per banana and the second lot at the rate of `4 for 5 bananas, his total collection would have been `460. Find the total number of bananas he had.
 - 12. A chemist has one solution which is 50% acid and a second which is 25% acid. How much of each should be mixed to make 10 litres of a 40% acid solution?
 - 13. A shopkeeper sold a saree and a sweater together for `1050, thereby making a profit of 10% on the saree and 25% on the sweater. If he had taken a profit of 25% on the saree and 10% on the sweater, he would have got `15 more. Find the cost price of each.
 - 14. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel

in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they mein one hour. What are the speeds of the two cars?

15.3 men and 4 boys can do a piece of work in 14 days, while 4 men and 6 boys can do it in 10 days. How long would it take 1 boy to finish the work?

Quadratic Equation: Polynomial of degree 2. Standard Form $ax2+bx+c=0 a\neq 0$

- The quantity b²-4ac in the quadratic formula is called the **discriminant**.
- 1. If b²-4ac>0, then we will get two real solutions to the quadratic equation.
- 2. If b^2 -4ac=0, then we will get a double root to the quadratic equation.
- If b²-4ac<0, then we will get two complex solutions to the quadratic equation

Solve the following equations

- 1. $x^{2}+40=-14x$
- 2. y²+12y+36=0
- 3. $3x^2=2x+8$
- 4. $10z^2+19z+6=0$
- 5. $5x^2=2x$
- 6. 1/(x+1)=1-5/(2x-4)
- 7. x+3+3/(x-1)=(4-x)/(x-1)
- 8. $5x^3-5x^2-10x=0$
- 9. 25y²-3=0
- 10. $(2t-9)^2=5$
- 12. x²-6x+1=0
- 13. $2x^2+6x+7=0$
- 14. 3x²-2x-1=0
- 15. $x^2+2x=7$
- 16. 3q²+11=5
- 17. 7t²=6-19t
- 18. 3/(y-2)=1/y +3
- 19. 16x-x²=0
- 20. 1/2x²+x-1/10=0
- 21. 0.04x²-0.23x+0.09=0
- 22. 13x²+1=5x
- 23. 6q²+20q=3
- 24. 49t²+126t+81=0
- 25. (x+3)/(x-3) + (x-6)/(x+6) = 11/5
- 26. X²+0.9x+0.36.

Word Problem:

- 27. The sum and product of two digits is 10 and 24. Find the numbers.
- 28. The difference between squares of two numbers is 120.

The square of smaller number is twice the greater number.

Find the numbers.

- 29. Find the value of a, for which one root of the quadratic equation $ax^2 14x + 8 = 0$ is six times the other.
- 30. Find *k* if x = 3 is a root of equation $kx^2 10x + 3 = 0$
- 31. In equation $x^2 4kx + k + 3 = 0$, Sum of the roots of a equation is double their product. Find k.
- 32. Product of Parul's age 2 years ago and 3 years hence is 84. Find her present age.
- 33. The sum of squares of two consecutive natural numbers is 244; find the numbers.

- 34. In the garden there are 150 mango trees. The number of trees in each row is 5 more than that in each column. Find the number of trees in each row and each column.
- 35. Find m if $(m 12) x^2 + 2(m 12) x + 2 = 0$ has real and equal roots.
- 36. Vivek is older than Kishor by 5 years. The sum of the reciprocals of their ages is 1616. Find their present ages.
- 37. Mr. Kasam runs a small business of making earthen toys. He makes certain number of toys on daily basis. Production cost of each toy is Rs 40 more than 10 times total number of toy, he makes in one day. If production cost of all toys per day is Rs 600, find production cost of one toy and number of toys he makes per day.
- 38. Pratik takes 8 hours to travel 36 km downstream and return to the same spot. The speed of boat in still water is 12 km. per hour. Find the speed of water current.
- Pintu takes 6 days more than those of Nishu to complete certain work. If they work together they finish it in 4 days. How many days would it take to complete the work if they work alone.
- 40. The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.
- 41. Two cars start out at the same point. One car starts out driving north at 25 mph. Two hours later the second car starts driving east at 20 mph. How long after the first car starts traveling does it take for the two cars to be 300 miles apart?