QUADRATIC EQUATION

(Ref: FM-QAH2022019)

TYPE I: Solving and forming quadratic equation

1. (a) Solve: $x^2 - 5x + 6 = 0$
   (b) Solve: $x^2 - x - 6 = 0$.

2. (a) Find the equation whose roots are 3 and 4.
   (b) Find the quadratic equation having rational coefficients and one root as $2 - \sqrt{3}$.

3. Form the cubic equation whose roots are 1, 2 and 3.

TYPE I: Sum & Product of roots

4. Find the value of k
   (a) If Roots of equation $2x^2 - 5x + (7k + 3) = 0$ are reciprocals of each others.
   (b) $2x^2 - (5k + 4)x + 7 = 0$ are equal in magnitude but opposite in sign.

5. If p and q are the roots of $x^2 - 3x - 5 = 0$ then find
   (a) $\frac{p}{q} + \frac{q}{p}$
   (b) $p^3 + q^3$

6. The equation $x^2 + 5x + 2 = 0$ has roots α and β. Find the equation whose roots are $\left(1 + \frac{\alpha}{\beta}\right)$ and $\left(1 + \frac{\beta}{\alpha}\right)$.

7. If α, β and γ are the roots of the equation $x^3 - 2x^2 + 3x - 6 = 0$, the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is

8. If p, q, and r are roots of $2x^3 - 3x^2 - x - 1 = 0$. Final $(1 - p)(1 - q)(1 - r)$.

9. If p, q, and r are roots of $x^3 - 5x - 4 = 0$. Find the value of $\frac{1}{p+q} + \frac{1}{q+r} + \frac{1}{r+p}$.

10. If a quadratic function have roots 3 & 4 and f(1) = 5. Find f(10).

11. If 3, 4, 5 are roots of cubic function, when co-efficient of x^3 is equal to 3. Find the value of function at x = 1.

12. "A" is a single-digit prime number and "B" is a natural number. How many equations of the form $x^2 - 4\sqrt{A}x + 3B = 0$ will have real roots?

TYPE III: Change of Roots

13. (a) Find the equation whose roots are one more than the roots of the equation $x^2 + 12x + 21 = 0$.

    (b) Find the equation whose roots are one less than the roots of the equation $x^2 + 12x + 21 = 0$.

    (c) Find the equation whose roots are twice the roots of the equation $x^2 + 12x + 21 = 0$.

    (d) Find the equation whose roots are half the roots of the equation $x^2 - 12x + 21 = 0$.

    (e) Find the equation whose roots are reciprocals of the roots of the equation $x^2 - 12x + 21 = 0$.

TYPE IV: Nature of roots

14. (a) Find the nature of the roots of the equation $3x^2 + 4x - 14 = 0$.
    (b) If a is real, what is the nature of the roots of $x^2 + 2(a + 1)x + 2a = 0$?

15. (a) If the roots of $x^2 - ax + 64 = 0$ are real, what value(s) can a take?
    (b) If the roots of $x^2 - ax + 25 = 0$ are not real, what value(s) can a take?

16. Equation $x^2 - 4x + c = 0$ has single roots/same root. Find c.

17. Equation $x^2 + 6x - c = 0$ has rational roots. Find least value of c.

TYPE V: Error Based

18. Ram finds roots of a Q.E as 3, 5 with mistake in x-coefficient and finds roots as 4, 10 with mistake in constant term. Find the actual equation.

TYPE VI: Common Roots

19. Equation $x^2 - x - 6 = 0$ & $2x^2 - kx + 3 = 0$ have a common root. Find k.

20. Equation $ax^2 + 2cx + b = 0$ and $ax^2 + 2bx + c = 0$ have exactly one common root. Find $a + 4(b + c)$.

21. Equation $x^2 + ax + 10 = 0$ and $x^2 + bx - 10 = 0$ have one common root. Find $a^2 - b^2$.

22. For $a \neq b \neq c$, if two equation $ax^2 + bx + c = 0$ and $(b + c)x^2 + (a + c)x + (a + b) = 0$ have both roots common, then find $a + c$.

23. Equation $x^2 - 3x + 9 = 0$ and $ax^2 + 12x + c = 0$ has a common root. Find $a + c$.

TYPE VII: Maxima/Minima

24. Find the minimum value of the expression $2x^2 + 8x + 5$. Also find the value of x, where the expression attains the minimum value.

25. A quadratic function attains maximum value of 3 at x = 1, value of same function at x=0 is 1. Find the value of function at x = 10.

26. If quadratic attains minimum value at x = 7 and f(7) = -16. If p, q are roots of equation. Find $|p - q|$.

TYPE VIII: Factor and Remainder

27. Q(x) is a Q.E. and Q(0) = 14. The remainder of Q(x) when divided by x + 1 is 25. The remainder of Q(x) when divided by x – 2 is 10. Find Q(x).

28. $f(x) = ax^3 + 3x^2 – 4x + b$. If f(x) is divisible by $x^2 - 4$. Find a + b.

29. Find the remainder when x^{100} is divided by $x^2 - 3x + 2$.

30. Find the remainder when x^5 is divided by $x^3 – 4x$.

TYPE IX: Descartes Rule

31. Using Descartes' rule of signs, comment on the roots of $x^4 - 3x^3 – x^2 - 5x + 4 = 0$

32. If the equation $x^5 + 15x^4 + 85x^3 + 225x^2 + 274x + a – 119 = 0$ has exactly 5 negative roots, then the value of a can be
    a) 80 b) 90 c) 120 d) 95

33. If the equation $x^3 - 6x^2 - ax – 6 = 0$ has all positive roots, then the value of 'a' could be
    a) -11 b) 11 c) 10 d) 9

34. If p > 0, q > 0, the number of real roots of the equation $2x^7 – px^5 – 3x^4 – qx^2 + 7 = 0$ can be
    a) 4 b) 3 c) 5 d) 6

Mixed

35. If the roots of the equation $ax^2 + bx + c = 0$ are p & q, find the equation whose roots are p^2 and q^2.
    a) $a^2x^2 + (b^2 - 2ca)x + c^2 = 0$
    b) $a^2x^2 - b^2x – 2cax + c^2 = 0$
    c) $a^2x^2 - b^2x + 2cax + c^2 = 0$
    d) $a^2x^2 + b^2 + 2cax + c^2 = 0$

36. If one root of the equation $x^2 - 10x+16 = 0$ is half of one of the roots of $x^2 - 4Ax + 8 = 0$. The value of A such that both the equation have integral roots is

37. Find positive integral value(s) of A such that the equation $2x^2 + 8x + A = 0$ has rational roots
    a) 8 b) 4 c) 6 d) (a) or (c)

38. Two equations have a common root which is positive. The other roots of the equation satisfy $x^2 - 9x + 20 = 0$. The product of the sums of the roots of the two equations is 42. The common root is:

39. The value of b in the equation $x^2 + ax + b = 0$, where one of the roots of the equation is $(3 + \sqrt{2})$ and a and b are integers is

40. Find the value of A, so that one of the roots of $x^2 + 5Ax + 27 = 0$ is the square of the other root.
    a) -12/5 b) -7/5 c) -6/5 d) -11/5

41. If the roots of the quadratic equation $x^2 - ax + b = 0$ are two successive multiples of 5, then the value of $a^2 - 4b$ is

42. Find the respective values of α and m if the roots of the quadratic equation $27x^2 - 87x + m = 0$ are α and 8/3
    a) 5/9, 40 b) 2/9, 40
    c) 4/9, 72 d) 5/9, 24

43. Kunal sells apples in boxes of different sizes. Apples are priced at 6 Rs per apple up to 100 apples" For every additional 10 apples over the first '100 apples, the price for all the apples goes down by 50 paise per apple. What should be the size of the box, such that the fully-packed box would fetch the greatest?
    a) 140 b) 110 c) 120 d) 100

Answer Key

1. a) 2, 3 b) -2, 3	2. a) $x^2 - 7x + 12 = 0$ b) $x^2 - 4x + 1 = 0$	3. $x^3 - 6x^2 + 11x – 6 = 0$	4. a) -1/7 b) -4/5	5. a) -19/5 b) 72
6. $2x^2 - 25x + 25 = 0$	7. ½	8. -1.5	9. 5/4	10. 35
11. -72	12. 21	13. a) $X^2 + 10x + 10 = 0$ b) $X^2 + 14x + 34 = 0$ c) $x^2 + 24x + 84 = 0$ d) $4x^2 - 24x + 21 = 0$ e) $21x^2 - 12x + 1 = 0$	14. a) real b) real	15. a) a > 16 or a < -16 b) -10 < a < 10
16. c = 4	17. 7	18. $x^2-14x+15 = 0$	19. 7 or -5.5	20. 0
21. 40	22. -b	23. -40	24. Min = -3, at x = -2	25. -159
26. 8	27. $3x^2 - 8x + 14 = 0$	28. -11	29. $(2^{100}-1)x+(2-2^{200})$	30. 16x
31.	32. C	33. A	34. B	35. C
36. 3/2	37. D	38. 2/3	39. 7	40. A
41. 25	42. A	43. B		