FUNCTIONS & GRAPHS

(Ref: FM-QAH2022022)

Sets

1. The set builder form of the set A = {5, 10, 15, 20, 25, 30} is
   a) {x/x is a multiple of 5 less than 30)
   b) {x/x is a multiple of 5 less than or equal to 30}
   c) (x/x is a multiple of 5 less than 30}
   d) {x/x is a multiple of 5)

2. The roster form of the set A = {x/x is an odd prime number less than 25) is
   a) {7, 11, 17, 19, 21}
   b) {3, 5, 19, 23}
   c) {3, 5, 7, 11, 13, 17, 19, 23}
   d) {3, 5. 7, 11, 13, 17)

3. The number of elements in the set {5, {2, 4), {3, 6), 10, 11} is

4. Which of the following is a subset of the set {{3. 5), 1, 4)?
   a) {1, {3}) b) {1, 2, 4, 5)
   c) {3, 5, 7} d) ({2, 7), 1)

5. If P is the set of all the letters of the word "FUNCTIONS", then cardinality of P is

Direction for questions 9 to 12: These questions are based on the following information.
Let x = {a, b, c, d, e, f, g, h}

6. The number of proper subsets of x is

7. The number of subsets of x that contain exactly 4 elements is
   a) 2^4 b) 15 c) 70 d) 64

8. The number of subsets of x that contain at most 5 elements is
   a) 219 b) 237 c) 220 c) 221

9. The number of subsets of x that contain b, c but not d is
   a) 16 b) 8 c) 32 d) 64

Functions

I) Domain and Range

10. f_1 = {(1, 1), (2, 1), (3, 1), (4, 1)} is f_1 a function on {1, 2, 3, 4}?

11. f_2 = {(1, 1), (1, 2), (2, 1), (2, 2)} is f_2 a function on {1, 2}?

12. Find the domain and range of the function f = {(1, a), (2, b), (3, c), (4, c)}.

13. Find the domain and range of the function f(x) = x^2.

14. Find the domain of the function $f(x) = \log(2x - 3) + \sqrt{x^2 - 9} + \frac{1}{\log(x-4)}$.

15. Find the range of the function $f(x) = \frac{x^2}{1+x^2}$, as x takes real values.

16. The range of the function $f(x) = \frac{3x+2}{|3x+2|}$, $x \neq -\frac{2}{3}$ is
    a) {-1, 1} b) {0, 1} c) {1, 2} d) {2, -1}

17. Find the domain of
    a) $f(x) = \sqrt{x-4} + \sqrt{x-8}$
    b) $f(x) = \frac{1}{\sqrt{|x|-x}}$

18. If $f(x) = \frac{3}{\sqrt{x(x-3)(x+2)}}$ is a real valued function then the domain of f(x) is
    a) (-2, 0) b) (-2, 0) U (3, ∞)
    c) (-∞, -2) d) (3, ∞)

19. Find the domain of the function $f(x) = \log|x| + \frac{1}{x+3}$
    a) R b) R – {0, 3}
    c) R – {0} d) R – {0, -3}

20. The domain of the function $f(x) = \log|2x^2 - 11x - 30|$ is
    a) $R - \left\{\frac{15}{2}, -2\right\}$ b) R – {-2}
    c) R d) $\left(-2, \frac{15}{2}\right)$

II) TYPES of functions:

21. Show that f(x) = 2x - 3 is a one-one function.

22. The number of one-one functions from set A to set B, where n(A) = 5 and n(B) = 8 is

23. Sets A and B are equivalent sets and n(A) = 6. How many bijections can be defined from B to A?

24. For two sets A and B, n(A) = 4, n(B) = 3. How many functions from A to B are neither onto nor constant?

25. For any function f(x) the function defined by $\frac{f(x)+f(-x)}{2}$ is ____.
    a) even b) odd c) Bothe even and odd d) None of these

26. The function which is both even as well as odd is __.
    a) Constant function
    b) Zero function
    c) Bijective function
    d) Sujective function

III) Operation on real function

27. $f(x) = 3x + 2, g(x) = 2x - 5$ then $fg(-2), (f^2 + g)(0)$ respectively are _____.

28. (a) $f(x) = x^2 - 3x + 2$ and $g(x) = 2x^2 - 5x + 2$. The domain of $\sqrt{f/g}$ is _____.
(b) $f(x) = 5x^2 - 2x + 28$ and $g(x) = x^2 + 4x - 36$. Find the domain of the function $\frac{f+g}{f-g}$.

IV) Inverse of a function

29. Find the inverse of the function $f(x) = 7x + 3$.

30. If $f(x) = \frac{2x-3}{3x+1}\left(x \neq -\frac{1}{3}\right)$, then $f^{-1}(2)$ is _____.

31. If $f(4x - 5) = \frac{x+2}{x}$, then find $f^{-1}(3)$.

V) Composite Function

32. If $f(x) = \frac{1-x}{1+x}$ and $x \neq -1$, find $fof(x)$.

33. $f(x) = \frac{3x+4}{5x-3}$, find $fof(2)$.

34. $f(x) = \frac{2x-3}{x-1}$ and x ≠ 1. $f^{-1}of(2) =$ ___.

35. If f(x) = $8x^4$ and g(x) = $\sqrt[3]{f(x)}$, find the value of $\log_2$(fog (8)).
a) 24 b) 25 c) 23 d) 26

Some Standard Types

36. If $f(x) + 3f\left(\frac{1}{x}\right) = 2x + \frac{1}{x}$, find $f(2)$.

37. If $f(x) + f(1 - x) = 1$, find the value of $f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + \cdots + f\left(\frac{96}{97}\right)$.

38. If $f(1) + f(2) + \cdots + f(n) = n^2 f(n)$ for all n > 1, and f(1) = 36000 then f(8) = _____.

39. If $f(x) = \frac{x-1}{x+1}$, find $f^{100}(x)$.

40. If $f(x)$ is a function satisfying $f(x).f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65$ then find f(6).

41. If f(x) is a function satisfying f(x+y) = f(x).f(y) and f(4) = 3 then find f(-8).

42. If f(x) is a function satisfying f(x+y) = f(x)+f(y) and f(3) = 29 then find f(27).

43. A function is defined as $\frac{[f(x-1)+f(x+1)]}{2} = f(x)$ and $f(1) = 2, f(2) = 5$. Find $f(5)$.

44. A function is defined as $f(x) = f(x-1) + f(x+1)$ and $f(1) = 2$ and $f(2) = 5$, find $f(1205)$.

Mixed Function

45. $f(x) = 3x^3 + 2x^2 + x + \frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}$.
$find\ \frac{f(2)}{f\left(\frac{1}{2}\right)} + \frac{f(3)}{f\left(\frac{1}{3}\right)} + \cdots + \frac{f(9)}{f\left(\frac{1}{9}\right)}$

46. $f(x) = \frac{4^x}{2+4^x}$ find $f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + \cdots + f\left(\frac{96}{97}\right)$

47. $f(x) = 2f(x-1)$ if x is even (x > 1)
$= 3g(x-1)$ if x is odd (x > 1)
$g(x) = 2g(x-1)$ if x is even (x > 1)
$= 3f(x-1)$ is x is odd (x > 1)
$f(1) = g(1) = 5$
Find f(6).g(6) and f(3).g(3).

48. $f(x+y), x-y) = x.y$
Find the expression for $f(x, y)$.
a) $x^2 - y^2$ b) $4(x^2 - y^2)$
c) $(x^2 - y^2)/4$ d) $x^2 + y^2$

49. $f(2x + 1, 3y - 1) = x + y$
Find $f(3x - 1, 2y + 1)$
a) x + y
b) $\frac{3}{2}x + \frac{2}{3}y - \frac{1}{3}$
c) $\frac{3}{2}x + \frac{2}{3}y + \frac{1}{3}$
d) $\frac{3}{2}x + \frac{2}{3}y$

50. $f(a + b) = f(a).f(b)$, for every real values of a & b. If f(4) = 8, find value of $f(1).f(2).f(3). - - - f(16)$.

51. A function 'f' is defined for all real numbers x and y as f(x + y) = f(x) + f(y) – xy – 1. If f(1) = 1, find f(-1/2).

52. Find the sum of the following expressions:
a) $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \cdots + [\sqrt{50}]$
b) $\left[\frac{1}{4}\right] + \left[\frac{1}{4} + \frac{1}{50}\right] + \left[\frac{1}{4} + \frac{2}{50}\right] + \cdots + \left[\frac{1}{4} + \frac{45}{50}\right]$
c) $\left[\frac{1}{5} + 1\right] + \left[\frac{2}{5} + 2\right] + \left[\frac{3}{5} + 3\right] + \cdots + \left[\frac{50}{5} + 50\right]$

53. If x is a natural numbers and $\left[\frac{x}{8}\right] = \left[\frac{x}{11}\right]$, then find how many values can x take.

54. Let $f(x) = [$ $2;\ when\ x\ is\ a\ rational\ number,$
$-2;\ when\ x\ is\ an\ irrational\ number.$
Find the value of the expression $f(|\sqrt{2}|) + |f\sqrt{2}| + \sqrt{|f(2)|} + |\sqrt{f(2)}|$
a) √2 b) 2 c) -√2 d) 2√2

Graphs

I) Going to graph

55. Draw the graphs of the following
I. (a) $y = x^2 + 3x + 4$
(b) $y = x^3$

II. (a) $y = |x|$
(b) $y = -|x|$

III. (a) $y = |x + 3|$
(b) $y = |x - 5|$

IV. (a) $y = |x| + 2$
(b) $y = |x| - 1$

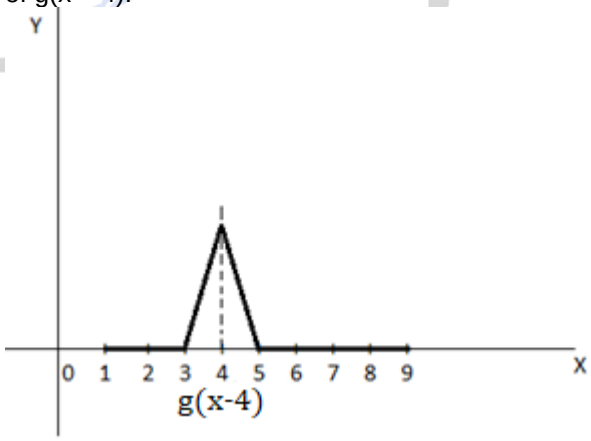
55. (a) Draw the graph of f(x) = [x]. where [x] represents the greatest integer less than or equal to x.
(b) Draw the graph of $f(x) = \frac{|x|}{x}$ for x ≠ 0,
= 0 for x = 0

(c) Draw the graph of y = [x] where [x] represents the least integer greater than or equal to x.

56. (a) Draw the graph of $f(x) = e^x$.
(b) Draw the graph f(x) = log x,
(c) Draw the graph of y = log (-x) for x < 0.

57. Draw the graph of $\frac{1}{x}, -\frac{1}{x}, \left|\frac{1}{x}\right|$.

58. The graph of f(x – 4) is shown below. Draw the graph of g(x – 4).



59. Draw the graph of $\log_e|x|$, $|\log_e x|$, $|\log_e|x|$, $\log_e(-x)$, $-\log_e x$, $e^{|x|}$, $|e^x|$, $e^{|x|}$, e^{-x}, $e^{-|x|}$

60. Draw the graph of $\left||x-1|+2\right|-3\right|$

61. Draw |x + 1|. |x – 2|

62. Find the number of points of intersection between
a) $f(x) = x^2 + 4x + 3$ and $g(x) = e^x$
b) $f(x) = x^2 - 4x - 3$ and $g(x) = e^x$
c) $f(x) = x^2 + 2x + 10$ and $g(x) = x$

II) Area enclosed

63. Find the area of region enclosed by the graphs of
a) $y = |x - 3| - 5 \& x - axis$
b) $y = 4 - \left|\frac{x}{2} - 3\right| \& x - axis$
c) $y = |2x - 3| - 5 \& x - axis$
d) $y = |x + 5| + 5 \& x - axis$
e) $f(x) = |x + 3| - 5, g(x) = 3 - |x + 5|$
f) $f(x) = |x|, g(x) = -|x| \& |x| = 5$
g) If the area enclosed by y = |2x| - a and y = a - |2x| is 36 sq. unit, find a.

64. Find the area of the region bound by $x^2 + y^2 > 2$ and $|x| + |y| \le 5$.

65. Find the area of |x + y| + |x – y| = 4.

66. What is the area of the Region enclosed by the graph of
a) |x| + |y| = 4
b) |x-1| + |y|= 4
c) |x + 12| + |y + 12| = 8
d) |y| - |x| = 3 and |y| = 7
e) |2x| + |3y| = 6

III) Number of Integral Solutions-

67. How many integral solutions are there of
1. |x| + |y| = 4
2. |x – 1| + |y – 1| = 5
3. |x + 2| + |y – 3| = 10
4. |x| + |y| ≤ 5
5. |x| + |y| < 5
6. |x – 1| + |y| ≤ 7
7. |x| + |y| ≤ 6 and y ≥ 0
8. a) y ≥ |x – 3| - 3 and y ≤ 0
b) y > |x – 3| - 3 and y ≤ 0
9. y ≤ 5 - |x – 1| & y ≥ 0

IV) Maxima/Minima

68. Find the minimum value of y
i. If y = max (x – 1, 3 – x)
ii. If y = max (5 – x, x – 1, 1)
iii. If y = max (x^2 - 3x, 5x – x^2)

69. Find the maximum value of y
I) If y = min (2x + 3, -x – 4)
II) If y = max (2x + 3, x – 1) & -10 ≤ x ≤ 10
III) If y = min (2x + 3, x – 1) & -10 ≤ x ≤ 10
IV) If y = min (5 – x, x – 1, 1
V) y = min (x^2 - 4, 4 – x^2)
VI) y = min (x^2 - 3x, 5x – x^2)

70. Find the maximum value of y, if y = min (|x – 2| - 2, 2 - |x – 2|)

Answer Key (Functions)

1. B
2. C
3. 5
4. D
5. 8
6. 255
7. C
8. A
9. C
10. Yes
11. No
12. {1,2,3,4}
13. Domain xϵR
    Range x ϵ [0, -∞)
14. (4,5)U(5,∞)
15. [0,1)
16. A
17. A) xϵ[4, 8]
    B) x < 0
18. b
19. d
20. a
21.
22. 6720
23. 720
24. 42
25. A
26. B
27. 36, -1
28. A) (-∞, ½) U [1,2) U {2, ∞)
    B) (-∞,∞)
29. (x-3)/7
30. -5/4
31. -1
32. X
33. 2
34. 2
35. C
36. 9/16
37. 48
38. 1000
39. X
40.
41. 1/9
42. 261
43. 14
44. -5
45. 8
46. 48
47.
48. C
49. B
50. 2^{102}
51. 5/8
52. A) 217
    B) 8
    C) 1510
53. 14 values
54. D
55.
56.
57.
58.
59.
60.
61.
62. a) 3
    b) 2
    c) 0
63. a) 25
    b) 32
    c) 12.5
    d) CBD
    e) 30
    f) 50
    g) ±6
64. 50 - 2π
65. 16
66. a) 32
    b) 32
    c) 128
    d) 32
    e) 12
67. i) 16
    ii) 20
    iii) 40
    iv) 61
    v) 41
    vi) 113
    vii) 49
    viii) a. 16, b. 9
    ix) 36
68. i) 1
    ii) 2
    iii) 0
69. I) +∞
    ii) 23
    iii) 9
    iv) 2
    v) 0
    vi) 4
70. 0