FUNCTIONS & GRAPHS

(Ref: FM-QAH2022022)

Sets

1. The set builder form of the set
   A - {4, 8. 12, 16, 20} is
   a) {x/x is a multiple of 4 less than 20)
   b) {x/x is a multiple of 4 less than or equal to 20}
   c) (x/x is a multiple of 4 less than 25}
   d) {x/x is a multiple of 4)

2. The roster form of the set A = {x/x is an odd prime number less than 20) is
   a) {7, 11, 17, 19, 2)
   b) {3, 5, 19, 23}
   c) {3, 5, 7, 11, 13, 17, 19}
   d) {2. 3, 5. 7, 11, 13, 17)

3. The number of elements in the set {5, {3. 6), {7, 8), 10, 11} is

4. Which of the following is a subset of the set {{3. 5), 1, 4)?
   a) {1, {3}) b) {1, 2, 3, 5)
   c) (3, 51 d) ({3, 5), 1)

5. If P is the set of all the letters of the word "MATHEMATICS", then cardinality of P is

6. If A = {1, 2, 3, 4} and B = {a, b, c}, then the number of elements in the cartesian product of A and B is

7. If n(A × B) = 48, then which of the following is not a value that n(A) can take?
   a) 12 b) 6 c) 14 d) 8

8. If a set A has 6 elements and set B has 4 elements. then the number of relations defined from A to B is
   a) 2^{16} b) 8^8 c) 2^{10} d) 8^{24}

Functions

I) Domain and Range

9. f_1 = {(1, 1), (2, 1), (3, 1), (4, 1)} is f_1 a function on {1, 2, 3, 4}?

10. f_2 = {(1, 1), (1, 2), (2, 1), (2, 2)} is f_2 a function on {1, 2}?

11. Find the domain and range of the function f = {(1, a), (2, b), (3, c), (4, c)}.

12. Find the domain and range of the function f(x) = x^2.

13. Find the domain of the function $f(x) = \log(2x - 3) + \sqrt{x^2 - 9} + \frac{1}{\log(x-4)}$.

14. Find the range of the function $f(x) = \frac{x^2}{1+x^2}$, as x takes real values.

15. The range of the function $f(x) = \frac{3x+2}{|3x+2|}$, $x \neq -\frac{2}{3}$ is
    a) {-1, 1} b) {1} c) {2, -2} d) R

16. Find the range of the function $f(x) = |x + 7| + |x - 9| + 12$
    a) (33, ∞) b) (12, ∞) c) (28, ∞) d) [28, ∞)

17. Find the domain of
    a) $f(x) = \sqrt{x-4} + \sqrt{x-8}$
    b) $f(x) = \frac{1}{\sqrt{|x|-x}}$

18. If $f(x) = \frac{3}{\sqrt{x(x-3)(x+2)}}$ is a real valued function then the domain of f(x) is
    a) (-2, 3) b) (-2, 0)∪(3, ∞)
    c) (4, ∞) d) (-∞, 0)

19. Find the domain of the function $f(x) = \log|x| + \frac{1}{x+3}$
    a) (-3, ∞) b) R – {0, 3}
    c) R – {0} d) R – {0, -3}

20. Find the domain of the function $f(x) = \sqrt{1 - \sqrt{1 - x^2}}$
    a) [0, 1] b) [-1, 1] c) [0, ∞) d) [-1, 0]

21. Find the range of the function $f(x) = |x - 2| - |x|, x \in R$.
    a) (-∞, 2] b) [-2, 2] c) [2, ∞) d) R

22. The domain of the function $f(x) = \log|2x^2 - 11x - 30|$ is
    a) $R - \left\{\frac{15}{2}, -2\right\}$ b) $\left(-2, \frac{15}{2}\right]$
    c) $\left(-2, \frac{15}{2}\right)$ d) $\left[-2, \frac{15}{2}\right)$

II) TYPES of functions:

23. Show that f(x) = 2x - 3 is a one-one function.

24. The number of one-one functions from set A to set B, where n(A) = 5 and n(B) = 8 is

25. Sets A and B are equivalent sets and n(A) = 6. How many bijections can be defined from B to A?

26. For any function f(x) the function defined by $\frac{f(x)+f(-x)}{2}$ is ____.
    a) even b) odd
    c) Both even and odd d) None of these

27. The function which is both even as well as odd is __.
    a) Constant function
    b) Zero function
    c) Bijective function

d) Identify function

III) Operation on real function

28. $f(x) = 3x + 2, g(x) = 2x - 5$ then $fg(-2), (f^2 + g)(0)$ respectively are _____.

29. (a) $f(x) = x^2 - 3x + 2$ and $g(x) = 2x^2 - 5x + 2$. The domain of $\sqrt{f/g}$ is ____.
(b) $f(x) = 5x^2 - 2x + 28$ and $g(x) = x^2 + 4x - 36$. Find the domain of the function $\frac{f+g}{f-g}$.

IV) Inverse of a function

30. Find the inverse of the function $f(x) = 7x + 3$.

31. If $f(x) = \frac{2x-3}{3x+1}\left(x \neq -\frac{1}{3}\right)$, then $f^{-1}(2)$ is ____.

32. If $f(x) = \frac{5x+3}{4x-9}$, $x \neq 9/4$, then $f^{-1}(x) =$
a) $\frac{9x+3}{4x-5}$ b) $\frac{5x+3}{4x-9}$ c) $\frac{9x-3}{4x+5}$ d) $\frac{5x-3}{9-4x}$

33. If $f(4x - 5) = \frac{x+2}{x}$, then find $f^{-1}(2)$.

V) Composite Function

34. If $f(x) = \frac{1-x}{1+x}$ and $x \neq -1$, find $fof(x)$.

35. $f(x) = \frac{3x+4}{5x-3}$, find $fof(x)$.

36. $f(x) = \frac{2x-3}{x-1}$ and x ≠ 1. $f^{-1}of(0) =$ ___.

37. If f(x) = $8x^4$ and g(x) = $\sqrt[3]{f(x)}$, find the value of $\log_2$ (fog (64)).
a) 39 b) 15 c) 28 d) 32

Some Standard Types

38. If $f(x) + 3f\left(\frac{1}{x}\right) = 2x + \frac{1}{x}$, find $f(2)$.

39. If $f(x) + f(1 - x) = 1$, find the value of $f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + \cdots + f\left(\frac{96}{97}\right)$.

40. If $f(1) + f(2) + \cdots + f(n) = n^2 f(n)$ for all n > 1, and f(1) = 36000 then f(8) = _____.

41. If $f(x) = \frac{x-1}{x+1}$, find $f^{100}(x)$.

42. If $f(x)$ is a function satisfying $f(x).f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(4) = 65$ then find f(6).

43. If f(x) is a function satisfying f(x+y) = f(x).f(y) and f(4) = 3 then find f(-8).

44. If f(x) is a function satisfying $f(x + y) = f(x) + f(y)$ and f(3) = 29 then find f(27).

45. A function is defined as $\frac{[f(x-1)+f(x+1)]}{2} = f(x)$ and $f(1) = 2, f(2) = 5$. Find $f(5)^2$.

46. A function is defined as $f(x) = f(x - 1) + f(x + 1)$ and $f(1) = 2$ and $f(2) = 5$, fid $f(1205)$.

Mixed Function

47. $f(x) = \frac{4^x}{2+4^x}$ find $f\left(\frac{1}{97}\right) + f\left(\frac{2}{97}\right) + \cdots + f\left(\frac{96}{97}\right)$

48. $f(x) = 2f(x - 1)$ if x is even (x > 1)
$= 3g(x - 1)$ if x is odd (x > 1)
$g(x) = 2g(x - 1)$ if x is even (x > 1)
$= 3f(x - 1)$ is x is odd (x > 1)
$f(1) = g(1) = 5$
Find f(6).g(6) and f(3).g(3).

49. $f(x + y,\ x - y) = x.y$
Find the expression for $f(x, y)$.
a) $x^2 - y^2$ b) $4(x^2 - y^2)$
c) $(x^2 - y^2)/4$ d) $x^2 + y^2$

50. $f(2x + 1, 3y - 1) = x + y$
Find $f(3x - 1, 2y + 1)$
a) x + y
b) $\frac{3}{2}x + \frac{2}{3}y - \frac{1}{3}$
c) $\frac{3}{2}x + \frac{2}{3}y + \frac{1}{3}$
d) $\frac{3}{2}x + \frac{2}{3}y$

51. Let $f(x) = \begin{cases} 2; \text{ when } x \text{ is a rational number,} \\ -2; \text{when } x \text{ is an irrational number.} \end{cases}$
Find the value of the expression $f(|\sqrt{2}|) + |f\sqrt{2}| + \sqrt{|f(2)|} + |\sqrt{f(2)}|$
a) √2 b) 2 c) -√2 d) 2√2

52.

x	1	2	3	4	5	6
g(x)	5	6	4	1	2	3

The table above defines g(x) for x = 1, 2, 3, 4, 5 and 6. For x > 6, g(x) = g(g(x-1). The value of g(899) is
a) 6 b) 5 c) 1 d) 2

Graphs

I) Going to graph

1. Draw the graphs of the following
   I. (a) $y = x^2 - 3x - 4$
   (b) $y = x^3$

   II. (a) $y = |x|$
   (b) $y = -|x|$

   III. (a) $y = |x - 3|$
   (b) $y = |x + 4|$

   IV. (a) $y = |x| + 4$
   (b) $y = |x| - 3$

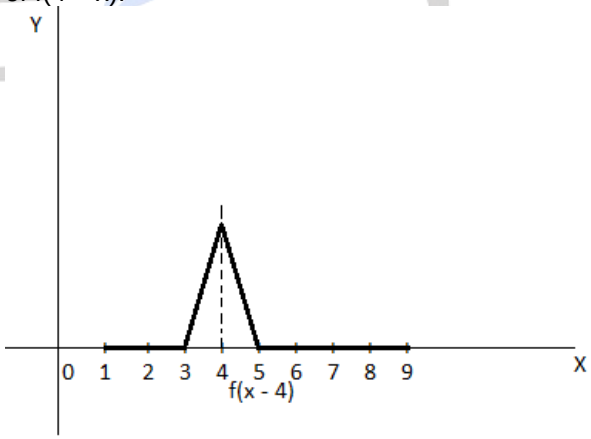
2. (a) Draw the graph of f(x) = [x]. Where [x] represents the greatest integer less than or equal to x.
   (b) Draw the graph of $f(x) = \frac{|x|}{x}$ for $x \neq 0$,

   (c) Draw the graph of y = [x] where [x] represents the least integer greater than or equal to x.

3. (a) Draw the graph of $f(x) = e^x$.
   (b) Draw the graph $f(x) = \log x$, $x \in R^+$.
   (c) Draw the graph of $y = \log(-x)$ for $x < 0$.

4. Draw the graph of $f(x) = \frac{1}{x}, g(x) = -\frac{1}{x}, h(x) = \left|\frac{1}{x}\right|$.

5. The graph of f(x – 4) is shown below. Draw the graph of f(4 – x).



6. Draw the graph of $\log_e|x|$, $|\log_e x|$, $|\log_e|x||$, $\log_e(-x)$, $-\log_e x$, $e^{|x|}$, $|e^x|$, $e^{|x|}$, e^{-x}, $e^{-|x|}$

7. Draw the graph of $\left||x-1|+2\right|-3\right|$

8. Draw |x – 1|. |x – 3|

II) Area enclosed

9. Find the area of region enclosed by the graphs of
   a) $y = |x - 3| - 5$ & $x - axis$
   b) $y = 4 - \left|\frac{x}{2} - 3\right|$ & $x - axis$
   c) $y = |2x - 3| - 5$ & $x - axis$
   d) $y = |x + 5| + 5$ & $x - axis$
   e) $f(x) = |x + 3| - 5, g(x) = 3 - |x + 5|$
   f) $f(x) = |x|, g(x) = -|x|$ & $|x| = 5$

10. Find the area bounded by $f(x) = |x|, g(x) = |x + 2|, |x| = 2$ with the x – axis.

11. Find the area of the region bound by $x^2 + y^2 > 2$ and $|x| + |y| \leq 5$.

III) Number of Integral Solutions-

12. How many integral solutions are there of
   I) $|x| + |y| = 4$
   II) $|x - 1| + |y - 1| = 5$
   III) $|x + 2| + |y - 3| = 10$

IV) Maxima/Minima

13. Find the real value of y
   i. If y = max (x – 1, 3 – x)
   ii. If y = min (5 – x, x – 1, 1)
   iii. If y = max (5 – x, x – 1, 1)
   iv. If y = max ($x^2 - 3x$, $5x - x^2$)

14. Find the maximum value of y
   I) If y = min (2x + 3, -x – 1)
   II) If y = max (2x + 3, x – 1) & $-10 \leq x \leq 10$
   III) If y = min (2x + 3, x – 1) & $-10 \leq x \leq 10$
   IV) If y = min (5 – x, x – 1, 1
   V) y = min ($x^2 - 4$, $4 - x^2$)
   VI) y = min ($x^2 - 3x$, $5x - x^2$)