# NUMBER SYSTEM

## Glossary

**Natural Numbers:** 1, 2, 3, 4.....
**Whole Numbers:** 0, 1, 2, 3, 4.....
**Integers:** ....-2, -1, 0, 1, 2 .....

**Rational Numbers:** Any number which can be expressed as a ratio of two integers for example a p/q format where 'p' and 'q' are integers. Proper fraction will have (p<q) and improper fraction will have (p>q)

**Factors:** A positive integer 'f' is said to be a factor of a given positive integer 'n' if f divides n without leaving a remainder. e.g. 1, 2, 3, 4, 6 and 12 are the factors of 12.

**Prime Numbers:** A prime number is a positive number which has no factors besides itself and unity.

**Composite Numbers:** A composite number is a number which has other factors besides itself and unity.

**Factorial:** For a natural number 'n', its factorial is defined as: n! = 1 x 2 x 3 x 4 x .... x n (Note: 0! = 1)

**Absolute value:** Absolute value of x (written as |x|) is the distance of 'x' from 0 on the number line. |x| is always positive. |x| = x for x > 0 OR -x for x < 0

**Concept:** The product of 'n' consecutive natural numbers is always divisible by n!

**Concept:** Square of any natural number can be written in the form of 3n or 3n+1. Also, square of any natural number can be written in the form of 4n or 4n+1.

**Concept:** Square of a natural number can only end in 0, 1, 4, 5, 6 or 9. Second last digit of a square of a natural number is always even except when last digit is 6. If the last digit is 5, second last digit has to be 2.

**Concept:** Any prime number greater than 3 can be written as 6k 1.

**Concept:** Any two digit number 'pq' can effectively be written as 10p+q and a three digit number 'pqr' can effectively be written as 100p+10q+r.

## Laws of Indices

- $a^m \times a^n = a^{m+n}$
- $a^m \div a^n = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^{\left(\frac{1}{m}\right)} = \sqrt[m]{a}$
- $a^{-m} = \frac{1}{a^m}$
- $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
- $a^0 = 1$

**Concept:** If $a^m = a^n$, then m = n

**Concept:** If $a^m = b^m$ and m ≠ 0;

Then a = b if m is Odd

Or a= b if m is Even

## Last digit of $a^n$

| n(Right) → a(Down)↓ | 1 | 2 | 3 | 4 | Cyclicity |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 4 | 8 | 6 | 4 |
| 3 | 3 | 9 | 7 | 1 | 4 |
| 4 | 4 | 6 | 4 | 6 | 2 |
| 5 | 5 | 5 | 5 | 5 | 1 |
| 6 | 6 | 6 | 6 | 6 | 1 |
| 7 | 7 | 9 | 3 | 1 | 4 |
| 8 | 8 | 4 | 2 | 6 | 4 |
| 9 | 9 | 1 | 9 | 1 | 2 |

**Concept**: The fifth power of any number has the same units place digit as the number itself.

## Last 2 digits

Last two digits of $a_1^b$ will be [last digit of a×b] 1

Last two digits of $24^{Odd}$ will be 24 and $24^{Even}$ will be 76. We can use this to find last two digits of any even number.

Last two digits of $a_5^x$, given that a is even, will be 25 if a is even. Last two digits of $a_5^x$, given that a is odd, will be 25 if x even and 75 if x id odd.

## HCF and LCM

For two numbers, HCF x LCM = product of the two.

HCF of fraction = $\frac{HCF\ of\ numerator}{LCM\ of\ denominator}$

**Relatively Prime or Co-Prime Numbers**: Two positive integers are said to be relatively prime to each other if their highest common factor is 1.

## Factor Theory

If N = $x^a y^b z^c$ where x, y, z are prime factors. Then, Number of factors of N =P=(a+1) (b+1) (c+1)

Sum of factors of N= $\frac{x^{a+1}}{x-1} \times \frac{y^{b+1}}{y-1} \times \frac{z^{c+1}}{z-1}$

Number of ways N can be written as product of two factors=$\frac{P}{2}$ or $\frac{P+1}{2}$ if P is even or odd respectively.

The number of ways in which a composite number can be resolved into two co-prime factors is $2^{m-1}$, where m is the number of different prime factors of the number.

Numberofnumbers which are less than N and co-prime to $\emptyset(N) = N\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)\left(1-\frac{1}{z}\right)$

**Concept:**If N= $(2)^a (y)^b (z)^c$ where x, y, z are prime factors Number of even factors of N =(a)(b+1)(c+1)
Number of odd factors of N = (b+1) (c+1)

## Divisibility Rules

A number is divisible by:

- 2, 4 & 8 when the number formed by the last, last two, last three digits are divisible by 2, 4 & 8 respectively.
- 3 & 9 when the sum of the digits of the number is divisible by 3 & 9 respectively.

- 11 when the difference between the sum of the digits in the odd places and of those in even places is 0 or a multiple of 11.
- 6, 12 & 15 when it is divisible by 2 and 3, 3 and 4 & 3 and 5 respectively.
- 7, if the number of tens added to five times the number of units is divisible by 7.
- 13, if the number of tens added to four times the number of units is divisible by 13.
- 19, if the number of tens added to twice the number of units is divisible by 19.

For ex., check divisibility of 312 by 7, 13 & 19

- **For 7:** $31 + 2 \times 5 = 31 + 10 = 41$ **Not divisible**
- **For 13:** $31 + 2 \times 4 = 31 + 8 = 39$ **Divisible.**
- **For 19:** $31 + 2 \times 2 = 31 + 4 = 35$ **Not divisible.**

## Algebraic Formulae

**$a^3 \pm b^3 = (a \pm b)(a^2\ ab + b^2)$.**
Hence, $a^3 \pm b^3$ is divisible by $(a \pm b)$ and $(a^2 \pm ab + b^2)$.

**$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + ... + b^{n-1})$**[for all n].
Hence, $a^n - b^n$ is divisible by $a - b$ for all n.

**$a^n - b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 ... - b^{n-1})$[**n-even]
Hence, $a^n - b^n$ is divisible by $a + b$ for even n.

$a^n + b^n = (a + b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 + ... + b^{n-1})$ [n-odd]
Hence, $a^n + b^n$ is divisible by $a + b$ for odd n.

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - ac - bc)$
Hence, *$a^3 + b^3 + c^3 = 3abc$ if $a + b + c = 0$*